

The Chain Rule

$$\frac{d}{dx} \{f[g(x)]\} = f'[g(x)] \cdot g'(x)$$

To take the derivative of a nested function, take the derivative of the outside function (keeping the inside function intact) and then multiply by the derivative of the inside function. You can keep the inside function intact by using a u-substitution. Example:

Find the derivative of $f(x) = 3\cos^4(x)$

The outside function is $3u^4$ and the inside function is $\cos(x)$. The derivative is:

$$12u^3 \cdot (-\sin(x))$$

Now replace u with $\cos(x)$ to get:

$$12\cos^3(x) \cdot (-\sin(x)) \text{ or} \\ -12\sin(x)\cos^3(x)$$