

Chapter 5 Review – graphs and their derivatives

1. f is continuous on $[0, 3]$ and satisfies the following

| | | | | |
|-------|---|----|----------------|----|
| x | 0 | 1 | 2 | 3 |
| f | 0 | 2 | 0 | -2 |
| f' | 3 | 0 | does not exist | -3 |
| f'' | 0 | -1 | does not exist | 0 |

| | | | |
|-------|-------------|-------------|-------------|
| x | $0 < x < 1$ | $1 < x < 2$ | $2 < x < 3$ |
| f | + | + | - |
| f' | + | - | - |
| f'' | - | - | - |

- Find the absolute extrema of f and where they occur.
- Find any points of inflection.
- Sketch a possible graph of f .

2. Sketch a smooth curve of $y = f(x)$ through the origin with the properties that

$$f''(x) < 0 \text{ for } x < 0 \text{ and } f''(x) > 0 \text{ for } x > 0.$$

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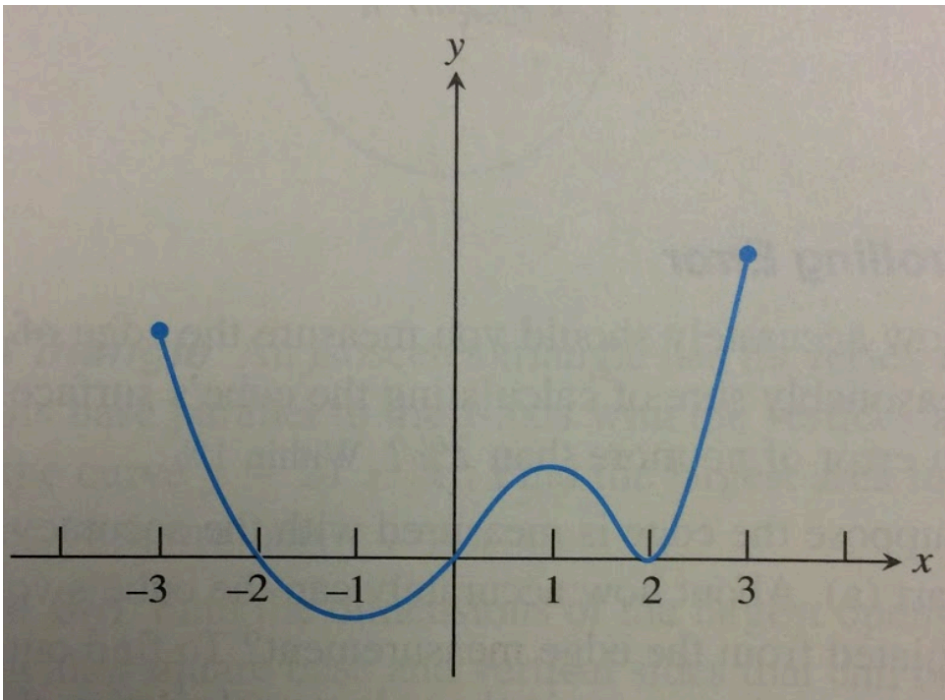
3. The accompanying figure shows the graph of the derivative of a function f . The domain of f is the closed interval $[-3, 3]$.

(a) For what values of x in the open interval $(-3, 3)$ does f have a relative maximum? Justify your answer.

(b) For what values of x in the open interval $(-3, 3)$ does f have a relative minimum? Justify your answer.

(c) For what values of x is the graph of f concave up? Justify your answer.

(d) Suppose $f(-3) = 0$. Sketch a possible graph of f on the domain $[-3, 3]$.



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4. The volume V of a cone is increasing at the rate of 4π cubic inches per second. At the instant when the radius of the cone is 2 inches, its volume is 8π cubic inches and the radius is increasing at $1/3$ inch per second.

(a) At the instant when the radius of the cone is 2 inches, what is the rate of change of the area of its base?

(b) At the instant when the radius of the cone is 2 inches, what is the rate of change of its height h ?

(c) At the instant when the radius of the cone is 2 inches, what is the instantaneous rate of change of the area of its base with respect to its height h ?

5. A piece of wire 60 inches long is cut into six sections, two of length a and four of length b . Each of the two sections of length a is bent into the form of a circle, and the circles are then joined by the four sections of length b to make a frame for a model of a right circular cylinder.

(a) Find the values of a and b that will make the cylinder of maximum volume.

(b) Use differential calculus to justify your answer in part (a).

